Nuclear Fusion

The Physics of Energy

Lecture # 14 a

Nuclear Fusion

- Introduction
- Lawson Criterion
- Confinement Mechanisms

Introduction

- The binding energy of ^4He ($E_b[^4He\ ] = 28.3$ MeV) is much larger than that of deuterium or tritium ($E_b[^2H\ ] = 2.22$ MeV ; $E_b[^3H\ ] = 8.48$ MeV) so their fusion reaction is exothermic:

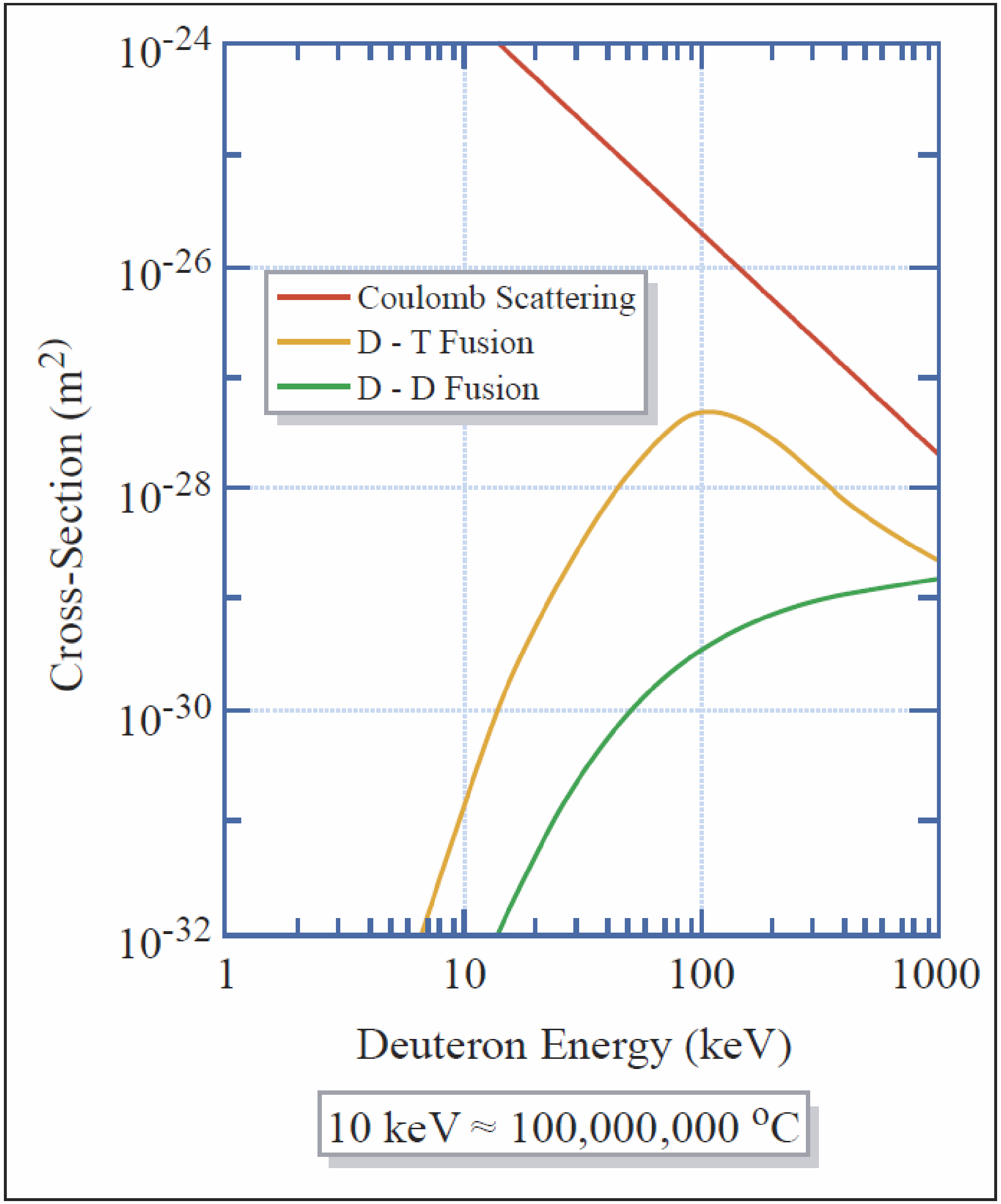
$$^2H + {}^3H \rightarrow {}^4He + n + 17.6 \text{ MeV}$$

- So are the deuterium fusion reactions:

$$^2H + {}^2H \rightarrow {}^3He + n + 3.27 \text{ MeV}$$

$$^2H + {}^2H \rightarrow {}^3H + p + 4.03 \text{ MeV}$$

- All of these reactions, however, have cross sections less than that of Coulomb scattering [Figure] being suppressed by the Coulomb barrier



Coulomb barrier suppression:

$$\exp- \left\{ \pi\alpha \sqrt{\frac{2\mu c^2}{E}} \right\}$$

with μ the reduced mass

- The exothermic nature of these reactions suggest that, besides nuclear fission, nuclear fusion may provide the basis for fueling a power plant
- Most of the interest has been focused in the D-T reaction, because it produces more energy and because its larger cross section peaks at lower energy
- The main disadvantages of using tritium, is that tritium must be produced since it does not exist in nature. However, it can be produced rather readily by neutron bombardment of ^{7}Li and ^{6}Li

- For a D- T fusion reactor to work, the deuterium and tritium need to be confined together for a sufficient long time τ_E in a plasma (ionized D^+, T^+ and e^-) with energy of O(10 KeV)
- To reach ignition the heat produced in the fusion reaction must be sufficient to keep the plasma temperature constant, so that the process can continue at the same rate
- A criterion was formulated about 50 years ago by John Lawson which gives the minimum conditions needed in the plasma to achieve ignition

Lawson Criterion

- To arrive at Lawson criterion, it is useful first to detail the energy per unit volume in a plasma of deuterium ions, tritium ions and electrons in equilibrium at a temperature T.
- From kinetic theory this energy density is:

$$e = \frac{3}{2} n_D k T + \frac{3}{2} n_T k T + \frac{3}{2} n_e k T$$

- It is natural to assume that the ion densities are equal and that

$$n_D = n_T = \frac{1}{2} n_e$$

- Then

$$e = 3 n_e k T$$

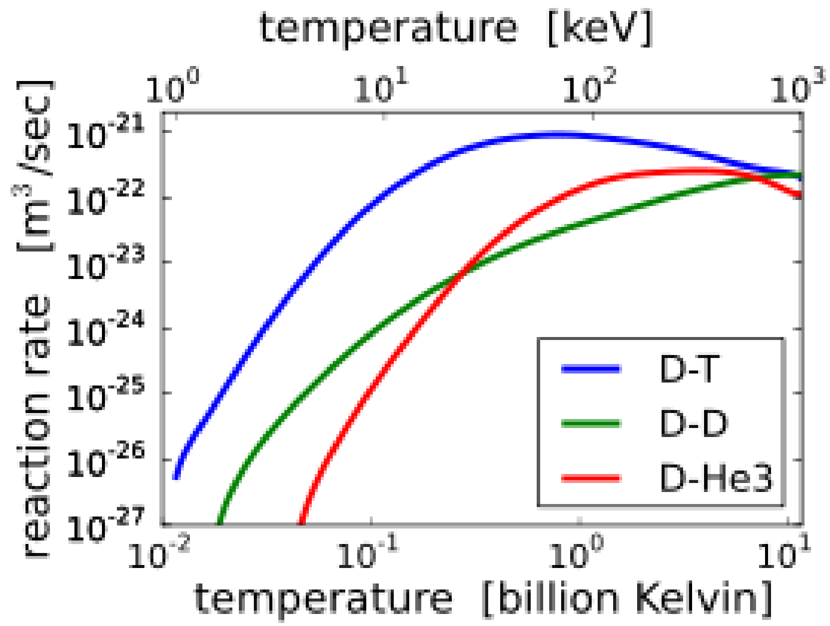
- If D-T fusion is occurring, part of the heat it produces will go on to heat the plasma. Let us calculate this heat by calculating the power per unit volume p_F produced in the reaction
- This power per unit volume is proportional to the number of fusions occurring per unit volume per unit time γ_F, given by

$$\gamma_F = n_D n_T <\sigma v> = \frac{1}{4} n_e^2 <\sigma v>$$

where $<\sigma v>$ is the D-T cross section averaged over a Maxwellian distribution of relative velocities at the temperature of the plasma T:

$$<\sigma v> = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \left(\frac{\mu}{kT}\right)^{\frac{3}{2}} \int dv\, v^3\, \sigma(v) \exp - \frac{\mu v^2}{2kT}$$

- The figure shows $<\sigma v>$ for various fusion reactions
- To compute the power per unit volume p_F produced one needs to multiply the fusion rate γ_F by the energy produced in the reaction $E = 17.6$ MeV :



$$p_F = \gamma_F E$$

- However, because neutrons do not contribute to reheat the plasma, the fraction contributing to reheating is:

$$p_R = \gamma_F E_\alpha ,$$

where $E_\alpha = 3.5$ MeV is the energy carried away by the α- particle produced in the D-T reaction,

- The criterion that Lawson enunciated for ignition comes from requiring that the power per unit volume produced by fusion which can reheat the plasma p_R must exceed the power loss per unit volume in the plasma from all sources p_{loss}
- If this is satisfied, then some of the excess power produced by fusion could be used to produce electricity.
- So the Lawson criterion is a criterion for the feasibility of fusion reactors
- The power loss density p_{loss} can be simply characterized by introducing a confinement time τ_E which measures the rate at which the plasma loses energy. Then

$$p_{loss} = e / \tau_E$$

- Thus the Lawson criterion is the requirement that:

$$p_R > p_{loss}$$

or

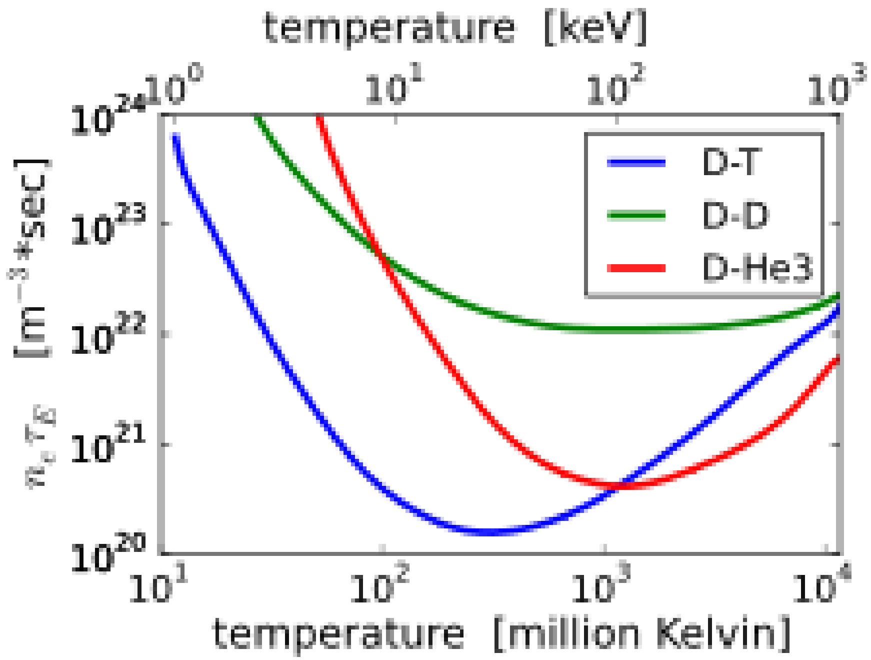
$$\gamma_F E_\alpha = \frac{1}{4} n_e^2 <\sigma v> E_\alpha > e / \tau_E = = 3 n_e k T / \tau_E$$

- One sees that the Lawson criterion gives a lower bound on the product of the electron density and the confinement time:

$$n_e \tau_E > \frac{12 k T}{<\sigma v> E_\alpha}$$

- The RHS above can be computed from a knowledge of $<\sigma v>$ and will have a minimum when $<\sigma v> / T$ is a maximum

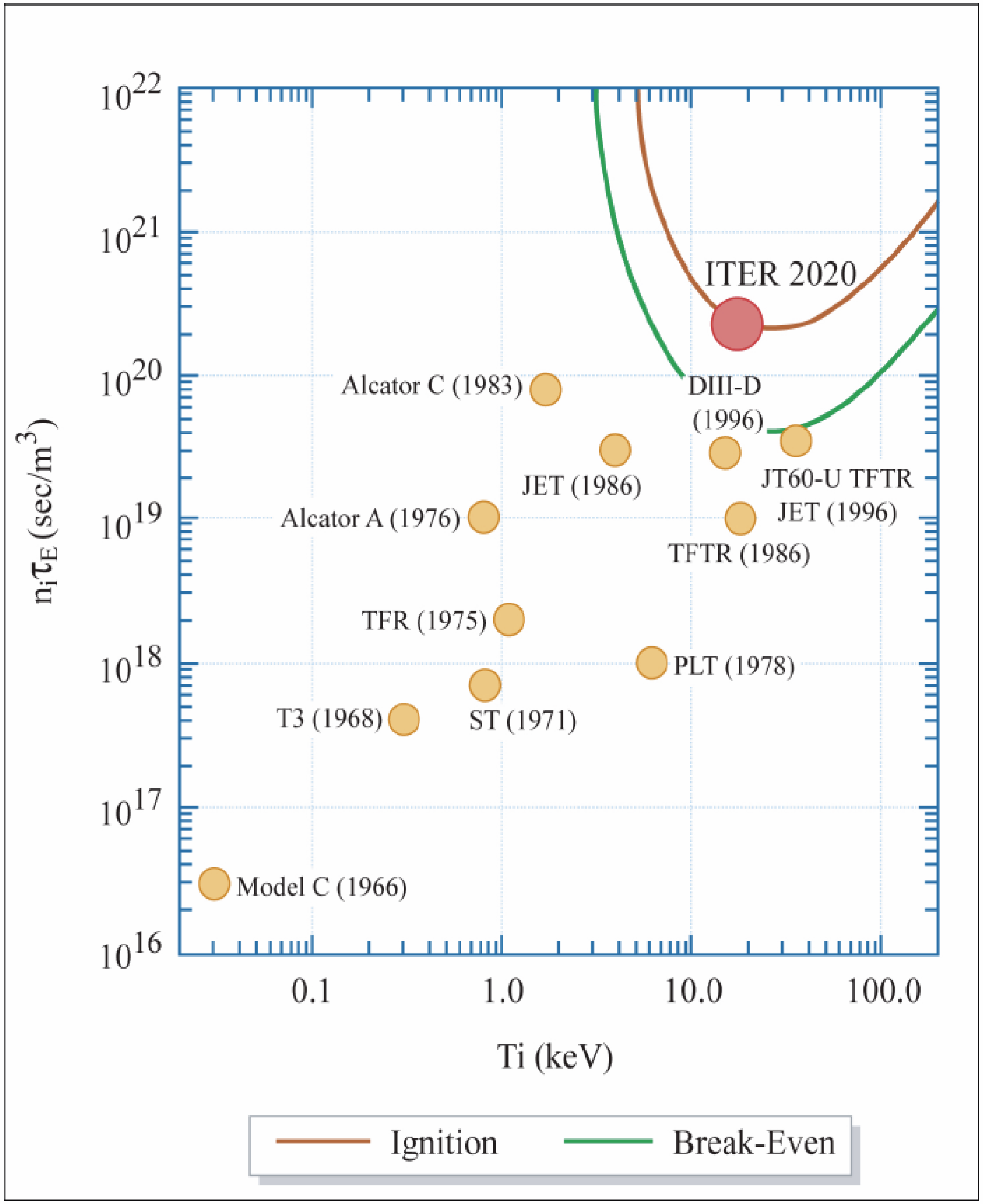
- The figure plots $n_e \tau_E$ versus T for various fusion reactions
- One sees, in particular, that for the D-T reaction the ignition limit is:



$$n_e \tau_E > 1.5 \times 10^{20} \text{ s/ m}^3$$ Lawson criterion

- Plasma densities of O($n_e \approx 10^{20}$ m^{-3}) are achievable so need confinement times of O($\tau_E \approx$ secs)
- Plot shows performance of different devices in Lawson space [Figure]

Most machines are still an order of magnitude away from needed $n_e\ \tau_E$ and operate at too low temperature T



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Confinement Mechanisms

- To achieve controlled fusion two types of plasma confinement mechanisms have been pursued:

i. Inertial confinement, in which you heat and compress the fuel to ignition before the constituents fly apart

ii. Magnetic confinement, where one uses the Lorentz force in a magnetic field to confine the charged particles in the plasma

- Both approaches are being pursued in two large facilities: the National Ignition Facility, at the Lawrence Livermore Lab, and ITER, the large international fusion reactor being built in France

- For inertial confinement powerful laser beams are used to implode small pellets containing a solid D-T fuel mixture
- The rapid heating caused by the laser pulse makes the surface of the pellet explode outwards , simultaneously imploding the fuel mixture to higher densities triggering fission
- For pellets of radius r= 0.1 mm, the confining time τ_E can be estimated simply as $\tau_E \approx r / v$, where v is the ion speed in the plasma which is of order

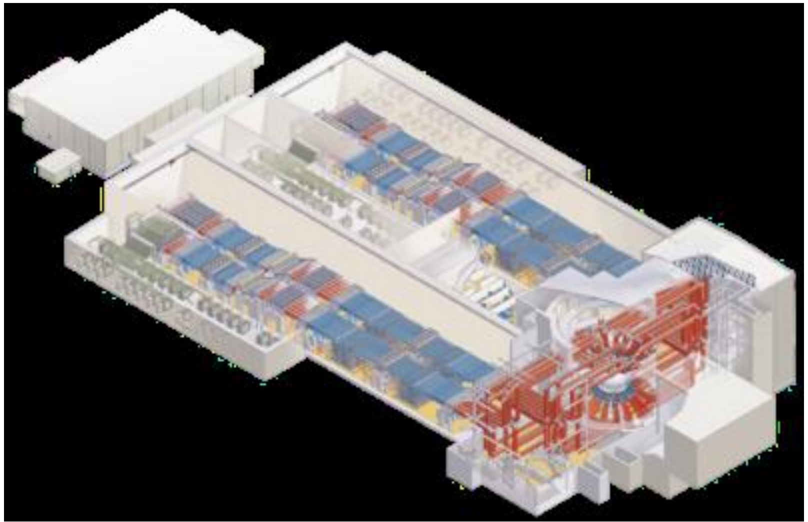
$v \approx \sqrt{2kT/m} \approx 2 \times 10^6$ m/s . Thus $\tau_E \approx 5 \times 10^{-11}$ s

- To satisfy Lawson's criterion for ignition, the ion density in the imploded pellet must reach the value

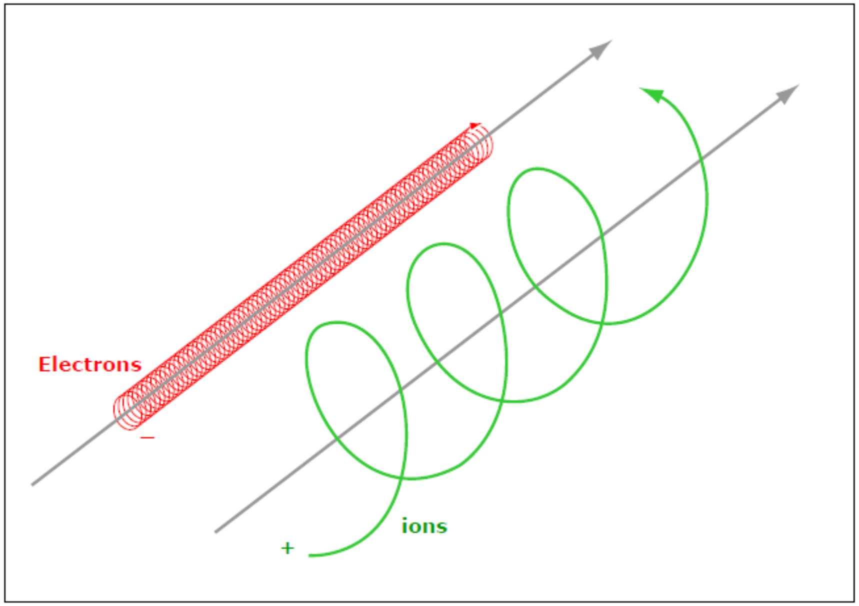
$$n \approx 10^{20} / \tau_E \approx 2 \times 10^{30}\ m^{-3}$$

which is two orders of magnitude above the density of the solid D-T pellets

- These densities have not yet been reached in the NIF facility

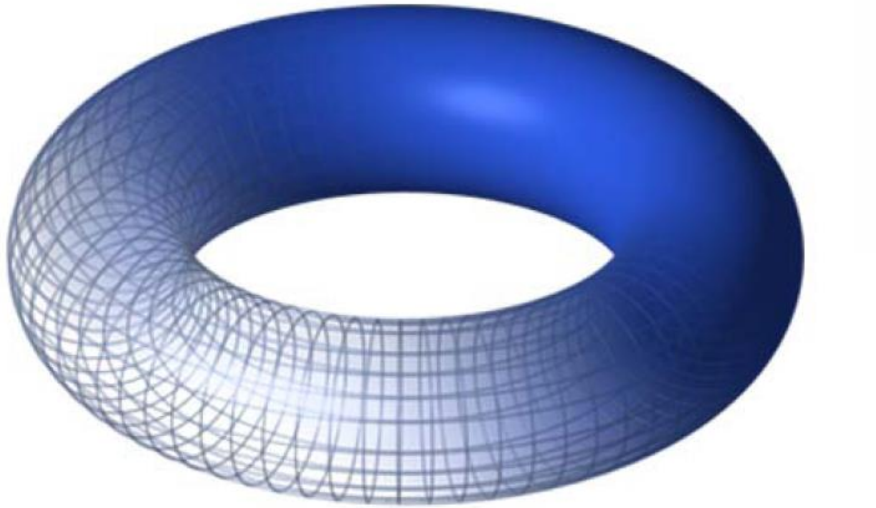


- For magnetic confinement, on the other hand, one uses the fact that charged particles in motion experience a Lorentz force **F = q(v x B)**
- The gyro radius $R = mv/qB$ at plasma energies of $kT = 10$ KeV and in a magnetic field $B = 5T$ are $R_D = 2.9$ mm and $R_e = 0.067$ mm



- This is not a confining structure, but can be made confining by using a toroidal structure- the Tokomak, suggested by Sakharov and Tamm

- Torus gets rid of edge effects



- More than 40-years of experience with tokomaks have led to ITER, now under construction
- Should get to Lawson criterion before 2025.
- Need to get to $\tau_E \approx 30$ sec